- A firm and a worker interact over 2 periods.

- Two types of worker:
  - Type $h$: produce $100$ for firm if hired
  - Type $l$: produce $10$ for firm

- Firm's prior is that prob of type $h$ is $p \in (0,1)$.

  And $100p + 10(1-p) > 15$

- Worker knows the type.

- Worker and firm are both risk neutral.

  => Maximize wages and profits over the two periods.

Game is:

Period 1:

- Firm offers a contract consisting of a probability of being hired $\alpha$.
  - Wage $w$ conditional on being hired.

- Worker accept or reject.

  If reject, firm gets $0$ and worker gets reservation wage $\$15$.
  If accept, then with prob $\alpha$ to be hired and receive $w$.

  With prob $(1-\alpha)$, the work is not hired and he and firm both receive $0$.

Period 2:

- If worker accepted and was hired in 1st period, his productivity is now known.
  - He receives the larger of his productivity or
reservation wage, while firm receives 0
\( * h \) receives $100 and \( l \) receives $15

- If worker did not accept or not accepted, his only option
  is to get $15, firm gets 0

(a) Suppose firm makes a contract offer that both \( h \) and \( l \)
  accept, what's its best offer? What's the profit?

(b) Suppose firm makes an offer that only \( h \) accepts. Assuming
    that the firm can set any w, including a negative one, what's
    the best offer? What's profit? Should offer this or (a)?

Answer:

(a) Firm's profit if both \( h \) and \( l \) accept:
\[
d [100p + 10(1-p) - w]
\]

IR constraints:

\( h \)
\[
d [w + 100] + (1-d) \cdot 15 \geq 30 \quad 1
\]

\( l \)
\[
d [w + 15] + (1-d) \cdot 15 \geq 30 \quad 2
\]

\[
\Rightarrow \text{Max } d [100p + 10(1-p) - w]
\text{ s.t. } 1 \quad 2\]
\[ 2W - 15 = 0 \quad \Rightarrow \quad W = 15 \]

For firm:

\[
\begin{align*}
\text{Max} & \quad \langle 100p + 10(1-p) - \frac{15}{d} \rangle \\
\Rightarrow \quad \text{Max} & \quad 90p + 10d - 15 \\
\text{Foc} & \quad 90p + 10 > 0 \quad \Rightarrow \quad \text{profit } \uparrow \text{ as } d \uparrow \quad \Rightarrow \quad d = 1
\end{align*}
\]

Thus, contract is like \( d = 1 \), \( W = 15 \), \( \pi_a = 90p - 5 \)

(b) Now only \( h \)-type accept

Firm's profit \( p \times [100 - w] \)

\[ \begin{align*}
\text{(IQ)} \\
\text{(h)} & \quad 2W + 85d - 15 > 0 \\
\Rightarrow \quad \text{must be bind} \quad \Rightarrow \quad W = \frac{15 - 85d}{d} \\
\end{align*} \]

Firm:

\[
\begin{align*}
\text{Max} & \quad p \times [100 - \frac{15 - 85d}{d}] \\
\Rightarrow \quad \text{Max} & \quad 185dp - 15p \\
\text{Foc} & \quad 185p > 0 \quad \Rightarrow \quad d \uparrow \text{ profit } \uparrow \quad \Rightarrow \quad d = 1
\end{align*}
\]

Thus \( W = -70 \), \( \pi_b = 100p - 15p + 85p = 170p \)

As \( 170p > 90p - 5 \) \quad \Rightarrow \quad \text{(b) contract is better}
2. Signaling.

- $ \Theta_L$ and $ \Theta_H$, (0.5, 0.5)

<table>
<thead>
<tr>
<th></th>
<th>if $e \leq 1$</th>
<th>if $e &gt; 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Theta_H$</td>
<td>100</td>
<td>110</td>
</tr>
<tr>
<td>$\Theta_L$</td>
<td>40</td>
<td>80</td>
</tr>
</tbody>
</table>

- Cost of education is $15 per unit for $ \Theta_H$ and $35 for $ \Theta_L$.

Find all separating and Pooling $E \Theta$ (pay productivity).

1. Separating $E \Theta$

- $e = 0$ not optimal for low-type now.
  
  If $e_i^* = 0$ $\Rightarrow$ $\bar{W}_l^* = 40$, $\bar{\pi}_l^* = 40$
  
  $e_i' = 1$ $\Rightarrow$ $\bar{W}_h \geq 80$, $\bar{\pi}_h \geq 45$

  $\Rightarrow$ deviating to $e_i = 1$

  $\Rightarrow$ $e_i^* = 1$ for this case.

For $ \Theta_H$ type

- if $e_i^* < 1$

  (IC) $100 - 15e_i^* \geq 80 - 15$ (not necessary)

  $100 - 35e_i^* \leq 80 - 35$ $\Rightarrow 35e_i^* \geq 55$

  $\Rightarrow e_i^* \geq 1$ $\Rightarrow$ O.W. be deviate.
2. Pooling $EX$

1. $e^* \in (0, 1)$

$$M(\Theta_H) = \frac{1}{2} \quad \Rightarrow \quad W^* = \frac{1}{2} 100 + \frac{1}{2} 40 = 70$$

1. $\frac{70 - 35e^*}{2} \geq 40 \quad \Rightarrow \quad e^* \leq \frac{6}{7}$

$$70 - 35e^* \geq 80 - 35 \quad \Rightarrow \quad e^* \leq \frac{5}{7}$$

1. $\frac{70 - 15e^*}{2} \geq 40 \quad \Rightarrow \quad e^* \leq 2$

$$70 - 15e^* \geq 80 - 15 \quad \Rightarrow \quad e^* \leq \frac{1}{3}$$

Also, $e^* = 0$ can hold =) $e^* \in [0, \frac{1}{3}]$. Pooling $\Theta_H$ exist if $M(\Theta_H) = \begin{cases} 1 \quad \text{if } e^* \geq e^* \\ 0 \quad \text{otherwise} \end{cases}$
\( e^x = 1 \)

let belief be \( u(\theta) = \begin{cases} \frac{1}{2} & \text{if } e^x \leq \theta \\ 0 & \text{otherwise} \end{cases} \)

\[ w^* = \frac{1}{2} \times 110 + \frac{1}{2} \times 80 = 95 \]

(i) \( 95 - 35 \geq 40 - 0 \) \( \Rightarrow \) Both hold

(ii) \( 95 - 15 \geq 40 - 0 \)

\( e^x > 1 \)

let belief be \( u(\theta) = \begin{cases} \frac{1}{2} & \text{if } e^x > \theta \\ 0 & \text{otherwise} \end{cases} \)

\[ w^* = 95 \]

(i) \( 95 - 35e^x \geq 40 \) \( \Rightarrow \) \( e^x \leq \frac{11}{7} \)

\( 95 - 35e^x \geq 80 - 35 \) \( \Rightarrow \) \( e^x \leq \frac{10}{7} \)

(ii) \( 95 - 15e^x \geq 40 \) \( \Rightarrow \) \( e^x \leq \frac{11}{3} \)

\( 95 - 15e^x \geq 80 - 15 \) \( \Rightarrow \) \( e^x \leq 2 \)

\( \Rightarrow e^x \in [1, \frac{12}{7}] \) can be pooling EAX