Mechanism Design Example

- 2 types of entrepreneurs, who are different only in term of how likely their project is to succeed
- Each has a project pays $S > 0$ if succeed
- Good entrepreneur has $P_h$ to succeed
- Bad  $P_i$ ($0 < P_i < P_h < 1$) to succeed
- Fraction $\lambda \in (0,1)$ of entrepreneurs are good
- To do the project, they must get a loan $L > 0$
- If he succeeds he gets $S - L$ interest and cannot default
- If he fails, can default to get $D > 0$, bank nothing
- If no loan, payoff is zero

Assume $S - L > D$

\[ P_i S + (1 - P_i) D - L > 0 \]

\[ \lambda P_h + (1 - \lambda) P_i < \frac{P_h L}{P_h S + (1 - P_h) D} \]

- Banks are risk neutral, so their expected payoff is zero
- If bank lend $L$ and get payback, gets $L(1+r)$
- If lead and get default, gets 0
- If bank doesn't lend, get L

(a) Suppose bank see the type of entrepreneur before setting the interest, what's the interest to offer? Under what condition entrepreneur take the loan? Information is complete, set interest rates contingent on types.

For high type \( (r_h) \)
\[
P_h L (1+r_h) - L = 0 \quad \text{(bank exp payoff = 0)}
\]
\[
\Rightarrow 1 + r_h = \frac{1}{P_h}
\]

For low type \( (r_l) \)
\[
P_l L (1+r_l) - L = 0
\]
\[
\Rightarrow 1 + r_l = \frac{1}{P_l}
\]

Will the entrepreneurs take the loan?

For high type:
\[
P_h \left[ s - (1+r_h) L \right] + (1-P_h) D \geq 0
\]
\[
\Rightarrow P_h s - L + (1-P_h) D \geq 0
\]

For low type:
\[
P_l \left[ s - (1+r_l) L \right] + (1-P_l) D \geq 0
\]
\[ P_L - L + (1 - P_L)D > 0 \] (satisfied by assumption)

As \( P_H > P_L \),
\[ P_H - L > 0 \]
\[ P_H - L + (1 - P_H)D > P_L - L + (1 - P_L)D > 0 \]

\[ \Rightarrow \text{both types will take the loan} \]

(b) Now the bank cannot see the types, what's the EQC outcome and how does it depend on \( \beta \)?

Now cannot discriminate, have to set a common interest rate.

Case 1: Only high type takes the loan.

As bank is risk neutral, set \( 1 + r = \frac{1}{P_H} \).

For low type:
\[ P_L [S - (1 + r) L] + (1 - P_L)D \]
\[ = P_L S + (1 - P_L)D - \frac{P_H}{P_L} L \]
\[ > P_L S + (1 - P_L)D - L > 0 \]

\[ \Rightarrow \text{low type will also take the loan} \Rightarrow \text{not EQC} \]

Case 2: Both types take the loan. (assume \( 1 + r = \frac{1}{P_L} \))

High type: \[ P_H S + (1 - P_H)D - \frac{P_H}{P_L} L \]

Low type: \[ P_L S + (1 - P_L)D - \frac{P_L}{P_L} L \]

For bank
\[ \nabla L \vec{p}_L \cdot (\vec{L} + (1-\lambda) \cdot \vec{C}_L \vec{p}_L) = \nabla L \vec{p}_L \cdot \vec{L} = 0 \]

\[ \Rightarrow \frac{1}{\vec{p}_L} = \frac{\nabla L}{\nabla \nabla L + (1-\lambda) \vec{p}_L \cdot \nabla L} = \frac{1}{\nabla \nabla L + (1-\lambda) \vec{p}_L} \]

For high type:

\[ P_h \cdot S + (1-P_h) D - \frac{P_h}{\nabla \nabla L + (1-\lambda) \vec{p}_L} < 0 \text{ by assump.} \]

\[ \Rightarrow x \in EO \]

Case 2: only low type take \( (1+r = \frac{1}{\vec{p}_L}) \)

as \( \frac{1}{\vec{p}_L} > \frac{1}{\nabla \nabla L + (1-\lambda) \vec{p}_L} \Rightarrow \text{high type not take} \)

by (a) low type takes \( \Rightarrow \text{only } EO \times \)