1. (1) 

\[
\begin{array}{c|cc}
& \text{St} & \text{Sw} \\
\text{St} & 0.0 & 4.1 \\
\text{Sw} & 1.4 & 3.3 \\
\end{array}
\]

Strictly D: None  
Weakly D: None  
Pure NE: (Sw, St) (St, Sw)

12. 

\[
\begin{array}{c|c|c}
& \text{Y} & \text{N} \\
\text{Alice} & 1,1,1 & 1,1,0 \\
\text{Bob} & 1,1,1 & 0,0,0 \\
\end{array}
\]

Strictly D: None  
Weakly D: (Alice) "No" (Bob) "No" (Chris) "Yes"  
Pure NE: (Yes, Yes, Yes) (Yes, Yes, No) (No, No, No)

13. Strictly D: None  
Weakly D: (Alice) "B" "C" (Bob) "A" (Chris) "A" "B"  
Pure NE: (A, A, A) (A, B, A) (B, B, B) (A, C, C) (C, C, C)
(4) Strictly D: None
Weakly D: None
Pure NE: (b, b) (g, e) (e, g)

2.
(1) EQ in strictly dominant strategy: None
EQ in weakly dominant strategy: 1st own value.

1. \( V_1 = 1, \ V_2 = 2 \)
   \( b = (1, 2) \)

2. \( V_1 = 1, \ V_2 = 4 \)
   \( b = (1, 4) \)

3. \( V_1 = 3, \ V_2 = 2 \)
   \( b = (3, 2) \)

4. \( V_1 = 3, \ V_2 = 4 \)
   \( b = (3, 4) \)

(2) If you follow the Grove Mechanism setting in the notes and use the summation of transfers as the definition of balanced budget, it will be fine.

Here, I use the actual payment as the base of transfer.
If we add the set of payments from $o_i$ to $i$ that depend only on the valuation of the other, 

$$t_i(b) = \begin{cases} -b_i + h_i(b) & \text{if } b_i > b_j \\ 0 + h_i(b) & \text{if } b_i < b_j \end{cases}$$

For 2 agents:

$$t_1(b_1, b_2) + t_2(b_1, b_2) = \begin{cases} -b_1 + h_1(b_2) + h_2(b) & \text{if } b_1 > b_2 \\ -b_1 + h_2(b_1) + h_1(b_2) & \text{if } b_1 < b_2 \end{cases}$$

Suppose the set of payments exist that will balance the budget.

$$\Rightarrow t_1(b_1, b_2) + t_2(b_1, b_2) = 0$$

Use the four cases we have:

1. $-1 + h_2(1) + h_1(2) = 0$  
2. $-1 + h_2(1) + h_1(4) = 0$  
3. $-2 + h_1(2) + h_2(3) = 0$  
4. $-3 + h_2(3) + h_1(4) = 0$

1 + 4 $\Rightarrow -4 + h_1(2) + h_1(4) + h_2(1) + h_2(3) = 0$

2 + 3 $\Rightarrow -3 + h_1(2) + h_1(4) + h_2(1) + h_2(3) = 0$

$\Rightarrow 3 = 4$  

Contradiction! Thus there is no such set of payments.
3.

(1) \[ \pi_i (V) = U_i (x_i (0), V_i) - t (x_i (0), \hat{V}) \]

(2) \[ \hat{V}_i = V_i \] is weakly dominant strategy.

Proof:

\[ \hat{V}_i^* \in \arg \max_{V_i} U_i (x^* (0), V_i) + \sum_{j \neq i} U_j (x^* (0), \hat{V}_j) - C (x^* (0)) - h_i (\hat{V}_i) \]

\[ \Rightarrow x^* \in \arg \max_{x \in X} U_i (x, V_i) + \sum_{j \neq i} U_j (x, \hat{V}_j) - C (x) - h_i (\hat{V}_i) \]

Same expression with

\[ x^* (V_i, \hat{V}_i) \in \arg \max_{x \in X} U_i (x, V_i) + \sum_{j \neq i} U_j (x, \hat{V}_j) - C (x) \]

\[ \Rightarrow \hat{V}_i = V_i \] is weakly dominant.

(3) Refer to notes.

(4) We can construct a NE such that player i always wins the object.

Let \[ s_i^* = \max_{R \in N} V_R \]

\[ s_j^* \leq 0, V_i, j \quad \forall j \in N \]

\[ s_i, s_j^* \leq 0, V_i, j \quad \forall j \in N \]

wts, this is NE.

(1) For player i:

\[ U_i (s_i, s_j^*) = V_i - \max_{j \neq i} s_j \geq 0 \]

a) if \[ s_i > s_i^* \] \[ \Rightarrow U_i (s_i, s_j^*) = V_i - \max_{j \neq i} s_j \geq 0 \]

b) if \[ s_i < s_i^* \text{ and } s_i > \max_{j \neq i} s_j \] \[ \Rightarrow U_i (s_i, s_j^*) = 0 \]

c) if \[ s_i < s_i^* \text{ and } s_i < \max_{j \neq i} s_j \] \[ \Rightarrow U_i (s_i, s_j^*) = 0 \]

\[ \Rightarrow \] No incentive to deviate
2. For player $j$ ($j \neq i$ and $j \in N$):

\[ U_j(s_j^*, s_j^*) = 0 \quad \text{ (as } j \text{ lost)} \]

a) if $s_j < s_j^* \Rightarrow U_j(s_j', s_j^*) = 0$

b) if $s_j' > s_j^*$ and $s_j' < s_i \Rightarrow U_j(s_j', s_j^*) = 0$

c) if $s_j' > s_j^*$ and $s_j' > s_i \Rightarrow U_j(s_j', s_j^*) = V_j - s_i < 0$

\Rightarrow \text{ No incentive to deviate}

\Rightarrow s^* = (s_i^*, s_j^*) \text{ is } NE \text{ and it is NOT in weakly dominant strategy.}

5. Refer to the notes.

4. (1)

<table>
<thead>
<tr>
<th>Cart</th>
<th>Bob</th>
</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>S</td>
</tr>
<tr>
<td>W</td>
<td>1, 1, 1, $-\frac{1}{2}$, $\frac{3}{2}$, 1</td>
</tr>
<tr>
<td>Alive</td>
<td>$\frac{1}{2}$, $\frac{3}{2}$</td>
</tr>
</tbody>
</table>

(2) Dominant strategy:

(Alive) : shirk

(Bob) : shirk

(Cart) : shirk

<table>
<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>W</td>
<td>S</td>
</tr>
<tr>
<td>W</td>
<td>$1, \frac{1}{2}, \frac{3}{2}$</td>
</tr>
<tr>
<td>Alive</td>
<td>$\frac{3}{2}$</td>
</tr>
</tbody>
</table>
Dominated strategies:
(Alice): Work
(Bob): Work
(Curt): Work

5.
(1) Normal form:
\[ N = \{1, 2, 5\} \]
\[ s_i \in [0, +\infty) \quad i \in N \quad \mathbb{S} = [0, +\infty) \times [0, +\infty) \]
\[ \pi_i = \max \{ 9_i (2 - 9_i - 8_j - c), -9_i : c \} \]

(2) Iterated elimination: refer to notes.

Graph:

Finally locate \((\frac{1}{3}, \frac{1}{3})\) to be EQR.

(3) the iteratively strictly undominated strategies are still
\[ S = \left( \frac{1}{3}, \frac{1}{3} \right) \]
\[ * \text{ since the process is the same} \]

NE: \[ S^* = \left( \frac{1}{3}, \frac{1}{3} \right) \text{ or } S^* = \left( \frac{1}{5}, \frac{1}{5} \right) \text{ where } 9_1 > 1 \text{ and } 9_2 > 1 \]

If WLOG, take player 1
\[ \text{if } s_i' < q_i \quad \Rightarrow \quad \pi_i = 0 \]
\[ \text{if } s_i' > q_i \quad \Rightarrow \quad \pi_i = 0 \quad \Rightarrow \text{No inactive to elevate} \]

6. \( N = \{1, 2, \ldots, n\} \)
\[ s_i = g_i = (0, 1) \quad \Rightarrow \quad s_i = \sum_{i=1}^{n} (0, 1) \quad \text{or} \quad s_i = \sum_{i=1}^{n} g_i \]
\[ u_i = u_i(x_i, G) = -u_i(1-g_i, g_i+G_i) \]

12) \( u_i(x_i, G) = x_i + \sqrt{G_i} \)
\[ u_i(g_i, g_i) = 1 - g_i + \sqrt{G_i} = 1 - g_i + \sqrt{g_i + g_i + G_i} \]
\[ \text{let } G_i = \sum_{j=1}^{n} g_j \]
\[ \Rightarrow u_i(g_i, G_i) = 1 - g_i + \sqrt{g_i + G_i} \]

In order to find best response \( \Rightarrow \text{FOC wrt } g_i \):
\[ \frac{\partial u_i}{\partial g_i} = -1 + \frac{1}{2 \sqrt{G_i}} = 0 \quad \Rightarrow \quad G_i = \frac{1}{4} \quad \Rightarrow \quad g_i = \frac{1}{4} - G_i \]

Then, the NE is: \( g^* = (g_1^*, \ldots, g_n^*) \) s.t. \( g_1^* + g_2^* + \ldots + g_n^* = \frac{1}{4} \)

13) \( u_i(x_i, G) = x_i + \frac{1}{4} \sqrt{G} \)
\[ u_i(g_i, G_i) = 1 - g_i + \frac{1}{4} \sqrt{G_i + G_i} \]

FOC (wrt \( g_i \))
\[ -1 + \frac{1}{2 \sqrt{G_i}} = 0 \quad \Rightarrow \quad G_i = \frac{1}{4} \]

Can this equation hold for all \( i \in N \)? \( \Rightarrow \) No.
For $i = n$  
$G^*(n) = \frac{1}{4n^2}$

For $i = n-1$  
$G^*(n-1) = \frac{1}{4(n-1)^2}$

... (5 equations continue)

For $i = 2$  
$G^*(2) = \frac{1}{16}$

For $i = 1$  
$G^*(1) = \frac{1}{4}$

For players with larger $i$, no incentive to invest more.

Suppose any of player with $i > 1$, $g_i > 0$, the most he/she can invest is $\frac{1}{4i^2} < \frac{1}{4}$  $\Rightarrow$ player 1 will invest more.

If player i reduce investment to zero, player 1 will still invest $\Rightarrow$ Incentive for player i to elevate.

$\Rightarrow$ Any strategy with $g_i > 0$ for $i > 1$ is NOT NE.

$\Rightarrow$ The only NE is $g^* = (1, 0, 0, \ldots, 0)$

(4)  
$U_i(x_i, G) = x_i \ln x_i + (1-x_i) \ln G$

$U_i(g_i, G-t) = x_i \ln (1-g_i) + (1-x_i) \ln (g_i + G-t)$

For C  
$-\frac{\partial U_i}{\partial g_i} + \frac{1-x_i}{G} = 0 \Rightarrow g_i = (1-x_i) - x_i \frac{1}{G}$

$\Rightarrow g_i (1-x_i) = (1-x_i) - x_i G$

$\Rightarrow g_i = 1 - x_i \frac{1}{1-x_i} G$

Sum the n equations  
$\sum g_i = n - \sum i \frac{x_i}{1-x_i} G$
\[ G = n - \epsilon_i \left( \frac{\partial}{\partial \epsilon_i} G \right) = n - G \epsilon_i \frac{\partial\epsilon_i}{\partial \epsilon_i} \]

\[ \Rightarrow \quad n = G \left( 1 + \epsilon_i \frac{\partial\epsilon_i}{\partial \epsilon_i} \right) \]

\[ G = \frac{n}{n + \epsilon_i \frac{\partial\epsilon_i}{\partial \epsilon_i}} \quad \text{(plug back to (\theta))} \]

\[ g_i = 1 - \frac{\partial\epsilon_i}{1-x^i} - \frac{n}{n + \epsilon_i \frac{\partial\epsilon_i}{\partial \epsilon_i}} = 1 - \frac{\partial\epsilon_i n}{(1-x^i) \left( 1 + \frac{n \partial\epsilon_i}{\partial \epsilon_i} \right)} \]

Thus, the NE is \( g^* = (g_1^*, \ldots, g_n^*) \) s.t.

\[ g_i^* = 1 - \frac{\partial\epsilon_i n}{(1-x^i) \left( 1 + \frac{n \partial\epsilon_i}{\partial \epsilon_i} \right)} \]