1. (30 points) Consider the following variant of the Prisoner’s dilemma. In period 1, each player picks either $C$ or $D$. The payoffs in the first period are given by

$$
\begin{array}{ccc}
\text{C} & \text{D} \\
\text{C} & 2 & 3 \\
\text{D} & -1 & 0 \\
\end{array}
$$

If $(C, D), (D, C)$ or $(D, D)$ is played in the first period, the game is over. If $(C, C)$ was played in the first period, the prisoner’s dilemma is played again in period 2. Again, if $(C, D), (D, C)$ or $(D, D)$ is played in the second period, the game is over, whereas if $(C, C)$ is played, then play moves to the third period. Continue the process recursively to time $T \leq \infty$ to define the dynamic game $G^T$. For the finite horizon case, assume that there is no discounting.

1. For $T = 3$, write down the reduced normal form of the game.
2. For $T = 3$ find all Nash equilibria of the game (don’t worry about mixed strategies).
3. For an arbitrary finite $T$ find all Nash equilibria of the game (don’t worry about mixed strategies).
4. For the case with an infinite horizon, assume that payoffs are discounted by factor $\frac{1}{\delta}$ with $\frac{1}{3} < \delta < 1$.

2. (30 points) Consider the world of peacocks. For simplicity we assume that all females are identical, so all a male peacock cares about is the probability of mating. Female peacocks on the other hand have a limited supply of eggs and want to mate only if the chance that the male has good genes is high enough. Suppose that a male peacock has good genes with probability $\alpha \in (0, 1)$. The (rational) male peacock then observes his genes and decides whether or not to grow long feathers or short feathers. Prior to mating the peacock may be eaten by a tiger and we assume that the probabilities of being eaten are

$$
\begin{array}{c|c|c}
\text{Good genes} & \text{Bad genes} \\
\hline
\text{Long feathers} & p & q \\
\text{Short feathers} & 0 & 0 \\
\end{array}
$$

where $p < q$. After the "tiger-eating peacocks-season" follows the mating season follows mating season. Conditional on eluding the tigers, the male peacock runs into a female who will always mate with probability $r$. However, with probability $1 - r$ the female peacock gets utility 0 if mating with a peacock with bad genes and 1 if mating with a peacock with good genes and $\frac{1}{2}$ if not mating. The male peacock on the other hand gets utility 1 if mating and 0 if not mating (it doesn’t matter whether he is eaten or not, all that matters is the probability of getting offspring).

1. Find a parametrization with pooling on short feathers being the unique equilibrium outcome.
2. Find a parametrization for which a separating and a pooling equilibrium both exist.
3. Can you find a parametrization where the peacocks with good genes randomize?

3. (40 points) Consider an environment with an indivisible public good and two players. Each player can be of type $\theta_i \in \{1, 4\}$ and the public good costs $C = 3$ to provide. Let $\alpha = \Pr[\theta_i = 4]$ and assume for now that types are independent. Unlike the case of a pure public good considered in class it is possible to exclude an agent from usage at no cost.

1. Define a direct revelation mechanism for this environment.
2. Find the ex post efficient allocation rule.
3. Construct a budget balancing mechanism that supports the ex post efficient allocation is a dominant strategy equilibrium.

4. Set up a planning problem that maximizes expected utility of the agents subject to incentive compatibility (as a Bayesian equilibrium), budget balance (in expectation) and individual rationality.

5. Find a condition on \( \alpha \) for when the first best efficient allocation rule solves the problem above and when the solution is inefficient from a first best perspective.

6. Instead assume that \( \theta_1 \) and \( \theta_2 \) are correlated with joint distribution

\[
\begin{array}{ccc}
\theta_2 = 1 & \theta_2 = 4 \\
\theta_1 = 1 & \beta & \gamma \\
\theta_1 = 4 & \gamma & \delta
\end{array}
\]

Can you now always support the ex post efficient allocation rule as a Bayesian equilibrium satisfying budget balance and individual rationality.