1. Consider a dynamic version of the “battle of the sexes” where Alice first decides whether to go to the opera or football and that Bob decides on opera or football after observing the choice of Alice. Assume that the payoffs are

\[ U_A(o, O) = 3, U_B(o, O) = 1 \]
\[ U_A(o, F) = 1, U_B(o, F) = 0 \]
\[ U_A(f, O) = 1, U_B(f, O) = 0 \]
\[ U_A(f, F) = 2, U_B(o, F) = 2 \]

1. Draw the extensive form.
2. Derive the normal form and the reduced normal form.
3. Find all Nash equilibria (pure and mixed).
4. Find all subgame perfect equilibria.

2. Consider an infinite repetition of

\[
\begin{array}{cc}
C & D \\
\hline
C & 2,2 & 0,4 \\
D & 4,0 & 1,1 \\
\end{array}
\]

and assume payoffs are discounted by \( \delta \in (0, 1) \). For \( \delta \) large enough, construct a subgame perfect equilibrium where \((C, c)\) is played in every period on the equilibrium path. Use Nash reversion in your construction and find a critical value for \( \delta \).

3. Consider a Cournot game with inverse demand \( p(y) = 1 - y \) and constant marginal cost \( c = 0 \). However, unlike what we did in class assume that there are three firms.

1. Find a Nash equilibrium such that \( y_1^* = y_2^* = y_3^* > 0 \) in the static Cournot oligopoly with three players.
2. Consider an infinite repetition with payoffs discounted by \( \delta \), find a condition on \( \delta \) for when it is possible for the three firms to divide the monopoly profit equally using a strategy with Nash reversion. Be careful to fully define the strategies.

4. Consider a bargaining game with 3 periods where in period 1 and 3 player 1 makes an offer and in period 2, player 2 makes an offer. Assume that player 1 discounts the payoffs by \( \delta_1 \) and player 2 discounts the payoffs by \( \delta_2 \). Also, assume that if player 2 rejects the offer from 1 in the last period the division

\[
\left( \frac{1 - \delta_2}{1 - \delta_1 \delta_2}, \frac{\delta_2 (1 - \delta_1)}{1 - \delta_1 \delta_2} \right)
\]

is implemented. The game is (like in class) that the player that is proposing can propose any division of a dollar and that the other player may either accept or reject. If the proposal is accepted the game is over and the proposed division is implemented. Otherwise either a new round of bargaining starts (if period 1 or 2 proposal is rejected) or the division above is implemented (if period 3 proposal is rejected).

1. Carefully explain what a strategy is for each player.
2. Find the backwards induction equilibrium.